Short Papers

Low-Noise Design of Microwave Transistor Amplifiers

R. S. TUCKER, STUDENT MEMBER, IEEE

Abstract—**Parameters are derived for circles of constant overall noise figure on the source admittance plane of a preamplifier cascaded with a noisy main amplifier. It is shown that the noise figure and noise measure of an amplifier can be expressed in terms of the scattering parameters of a lossless two-port network connected at the input of the amplifier. Examples are given which demonstrate how this network can be synthesized to meet amplifier noise specifications.**

I. INTRODUCTION

The theory of noise in linear two-port networks has been developed extensively in recent decades. It is known, for example, that the noise figure [1] and noise measure [2] of a two-port network can be minimized by appropriate selection of the source admittance and that loci of constant noise figure and constant noise measure are circles on the source admittance plane. Graphical constructions of this type are useful in the design of coupling networks for low-noise microwave transistor amplifiers [3] where the source admittance must be carefully controlled for optimum noise performance. The overall noise performance of a preamplifier cascaded with a noisy main amplifier has been investigated by Baechtold and Strutt [4] who have found the optimum source admittance for the preamplifier. Hartmann and Strutt [5] have presented general formulas for noise parameters of two-ports under a number of network transformations.

This short paper analyzes the amplifier considered by Baechtold and Strutt [4] and presents loci of constant overall noise figure on the source admittance plane. Furthermore, it is shown that a lossless two-port coupling network, connected at the input of an amplifier, can be designed to meet specifications on amplifier noise figure or noise measure by relating the noise figure of the amplifier to the transducer power gain of the network. Consequently, the graphical construction of noise figure or noise measure loci may be avoided.

II. THEORY

The amplifier considered here is shown in Fig. 1. A preamplifier is driven from the source admittance Y_s which is provided by the lossless two-port coupling network N. The preamplifier has noise figure F and available gain G_a and is cascaded with a main amplifier of noise figure F'.

Constant Noise Figure Loci

The overall noise figure of the amplifier in Fig. 1 is

$$F_t = F + \frac{E}{G_a} \tag{1}$$

where $E = F' - 1$ is the excess noise figure of the main amplifier. The noise figure of the preamplifier is given by [1]

$$F = F_{\min} + \frac{R_{ef}}{G_s}[(G_s - G_{of})^2 + (B_s - B_{of})^2] \tag{2}$$

Manuscript received October 21, 1974; revised February 10, 1975. This work was supported by the Australian Radio Research Board.
The author was with the Department of Electrical Engineering, University of Melbourne, Parkville, Vic., Australia. He is now with the Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, Calif. 94720.

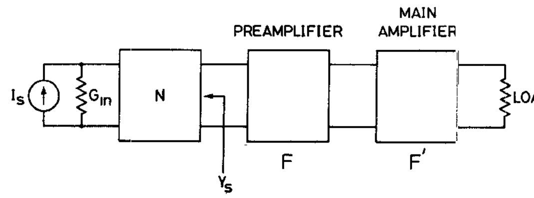

Fig. 1. The amplifier.

where $F_{\min}$ is the minimum noise figure of the preamplifier, R_{ef} is a parameter, and $Y_{of} = G_{of} + jB_{of}$ is the source admittance which produces $F_{\min}$. Similarly [2], the available gain of the preamplifier is given by

$$\frac{1}{G_a} = \frac{1}{G_{a_{\max}}} + \frac{R_{eg}}{G_s}[(G_s - G_{og})^2 + (B_s - B_{og})^2] \tag{3}$$

where $G_{a_{\max}}$ is the maximum available gain, R_{eg} is a parameter, and $Y_{og} = G_{og} + jB_{og}$ is the source admittance which produces $G_{a_{\max}}$. Loci of constant noise figure can be obtained by substituting (2) and (3) in (1) and rearranging. Hence

$$(G_s - G_{op})^2 + (B_s - B_{op})^2 = G_{RP}^2 \tag{4}$$

where

$$G_{op} = \frac{2G_{of}R_{ef} + 2G_{og}ER_{eg} + F_t - F_{\min} - \dfrac{E}{G_{a_{\max}}}}{2(R_{ef} + ER_{eg})} \tag{5}$$

$$B_{op} = \frac{B_{of}R_{ef} + B_{og}ER_{eg}}{R_{ef} + ER_{eg}} \tag{6}$$

and

$$G_{RP} = \frac{1}{2(R_{ef} + ER_{eg})}\left[\left(F_t - F_{\min} - \frac{E}{G_{a_{\max}}}\right)^2 + 4(G_{of}R_{ef} + G_{og}ER_{eg})\left(F_t - F_{\min} - \frac{E}{G_{a_{\max}}}\right) - 4R_{ef}R_{eg}E\{(G_{of} - G_{og})^2 + (B_{of} - B_{og})^2\}\right]^{1/2}. \tag{7}$$

In practice, E can be assumed independent of Y_s if the preamplifier is unilateral or if the main amplifier is adjusted for a fixed E by appropriate choice of a lossless coupling network connected between the preamplifier and main amplifier. Under these assumptions, (4) describes circles of constant overall noise figure on the source admittance plane with centers at $Y_{op} = G_{op} + jB_{op}$ and radii G_{RP} as shown in Fig. 2. As an illustration, in Fig. 3 circles are plotted on the Smith chart [2] for an 11-dB overall amplifier noise figure. The preamplifier uses an AT-561 transistor [6] with a collector current of 3 mA, a collector to emitter voltage of 10 V, and a frequency of 6 GHz. The circles are drawn with F' as a parameter, varying from 11 to 13.3 dB. For F' greater than 13.3 dB, an overall noise figure of 11 dB cannot be obtained.

In addition to describing the noise performance of a preamplifier cascaded with a noisy main amplifier, the foregoing results can be used to find the noise measure of a single amplifier stage. If the main amplifier of Fig. 1 comprises a large number of stages, each identical to the preamplifier, then the overall noise figure of the amplifier is

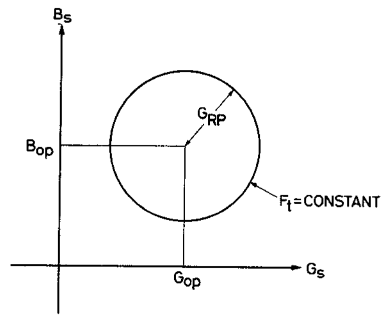

Fig. 2. Locus of constant overall noise figure on the source admittance plane.

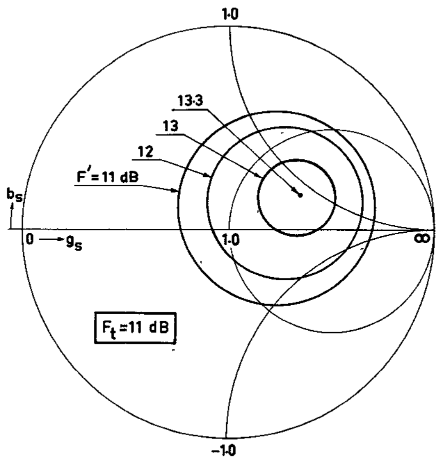

Fig. 3. Loci of constant overall noise figure on the Smith chart as a function of main amplifier noise figure. The preamplifier uses an AT-561 transistor and the chart is normalized to 20 mmho.

$$F_t = 1 + \frac{F-1}{1-1/G_a} = 1 + M \tag{8}$$

where M is the noise measure of each stage. For a large number of stages, the noise figure of the main amplifier is also equal to F_t. Thus

$$E = M. \tag{9}$$

Substituting (8) and (9) in (5) and (6), one obtains the centers of circles of constant noise measure as found by Fukui [2]. The radii of these circles[1] are obtained from (8) and (9) in (7).

Input Network Design

As was mentioned in the introduction, loci of constant noise figure and noise measure find application in the design of amplifier coupling networks. In the narrow-band case, a few noise figure loci on the source admittance plane are usually adequate to enable an appropriate design to be selected. With broad-band amplifiers, however, noise calculations are necessary at more than one frequency and a large number of circles may be required. The resulting design procedure can be quite cumbersome and tedious. An alternative approach is to directly synthesize the networks from noise figure specifications.

As the first step in a method of coupling network synthesis (4) is written in the form

$$F_t = F_{tm} + \frac{R_{ef} + ER_{eg}}{G_s}\left[(G_s - G_o)^2 + (B_s - B_o)^2\right] \tag{10}$$

where

$$F_{tm} = F_{\min} + \frac{E}{G_{a_{\max}}} + 2(R_{ef} + ER_{eg})G_o - 2(G_{of}R_{ef} + G_{og}ER_{eg}) \tag{11}$$

$$G_o = \frac{1}{R_{ef} + ER_{eg}}\left[(R_{ef} + ER_{eg})(G_{of}^2R_{ef} + G_{og}^2ER_{eg}) + R_{ef}ER_{eg}(B_{og} - B_{of})^2\right]^{1/2} \tag{12}$$

and

$$B_o = B_{op}. \tag{13}$$

The minimum overall noise figure of the amplifier is F_{tm} and is obtained when $Y_s = G_o + jB_o$, the optimum preamplifier source admittance. The minimum noise measure of a single-stage amplifier[1] $M_{\min}$ is obtained from (11) with $F_{tm} = 1 + M_{\min}$ and $E = M_{\min}$

$$M_{\min} = [M_2 + (M_2^2 - M_1M_3)^{1/2}]/M_1 \tag{14}$$

where

$$M_2 = \left(1 - \frac{1}{G_{a_{\max}}} + 2R_{eg}G_{og}\right)(F_{\min} - 1 - 2R_{ef}G_{of}) + 2R_{eg}R_{ef}(|Y_{og}|^2 + |Y_{of}|^2 - 2B_{og}B_{of}) \tag{15}$$

and M_1 and M_3 are as defined in [2].

The source admittance for minimum noise measure $Y_{om} = G_{om} + jB_{om}$ [2] can be obtained by substituting $E = M_{\min}$ in (12) and (13).

In Fig. 4, a lossless two-port input coupling network N with scattering matrix S is connected between the source conductance G_{in} and the admittance $Y_o^* = G_o - jB_o$ where the asterisk represents the complex conjugate and S is normalized to G_{in} at the input port and to Y_o^* at the output port. Generally Y_o^* is different from the input admittance of the preamplifier, but the connection of Fig. 4 enables the overall noise figure of the amplifier of Fig. 1 to

[1] There are errors in Fukui's formula [2] for this parameter.

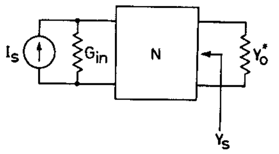

Fig. 4. Lossless coupling network.

be related to scattering parameters of N. This relationship is obtained by writing the inverse of the transducer power gain of N in the form

$$\frac{1}{|s_{21}(j\omega)|^2} = 1 + \frac{1}{4G_sG_o}[(G_s - G_o)^2 + (B_s - B_o)^2] \quad (16)$$

where

$$|s_{21}(j\omega)|^2 + |s_{11}(j\omega)|^2 = 1 \quad (17)$$

and by substituting (16) in (10)

$$|s_{21}(j\omega)|^2 = \frac{4G_o(R_{ef} + ER_{eg})}{(F_t - F_{tm}) + 4G_o(R_{ef} + ER_{eg})}. \quad (18)$$

The corresponding normalizing admittance at the output port of N is given by (12) and (13). For a single-stage amplifier $E = 0$ is substituted into (18) to obtain

$$|s_{21}(j\omega)|^2 = \frac{4\alpha}{F - F_{\min} + 4\alpha} \quad (19)$$

where $\alpha = G_oR_{ef}$ is the invariant noise parameter defined by Lange [7].

Substituting (8) and (9) in (18) and putting $F_{tm} = 1 + M_{\min}$, one obtains the transducer power gain as a function of the noise measure M

$$|s_{21}(j\omega)|^2 = \frac{4G_{opm}(R_{ef} + MR_{eg})}{(M - M_{\min}) + 4G_{opm}(R_{ef} + MR_{eg})}. \quad (20)$$

The corresponding normalizing admittance $Y_{opm}^* = G_{opm} - jB_{opm}$ is obtained by substituting (8) and (9) in (12) and (13). It should be noted that Y_{opm} is a function of M and is different from Y_{om} except when $M = M_{\min}$. In the following section, however, it will be seen that in a practical low-noise design problem where $M \simeq M_{\min}$ at all frequencies in the amplifier passband, Y_{opm} and Y_{om} are nearly identical.

III. DESIGN EXAMPLES

The two examples below use a GaAs Schottky-barrier-gate field-effect transistor which was described recently [3]. The design method is simpler and gives greater insight into the broad-band low-noise design problem than the alternative graphical technique which requires the construction of noise figure or noise measure loci.

First, consider an amplifier comprising identical cascaded stages with each stage designed on a near-minimum noise measure basis. It is assumed that the output of each transistor is coupled to the following transistor by a lossless two-port network and that transistors are the only source of amplifier noise. Using (20), a specification of $M = M_{\min}$ for each stage of the amplifier gives $|s_{21}|^2 = 1$ across the amplifier passband. This requirement of perfect match between the network N and the admittance Y_{opm}^* cannot be met, in general, due to the gain–bandwidth limitations imposed by Y_{opm}^* [9]. In order to achieve a realistic gain–bandwidth requirement of N, a 0.5-dB maximum increase from optimum overall amplifier noise figure is specified for the present problem.

If the number of amplifier stages is large, then from (8) the maximum allowable noise measure for each stage is

$$M = 1.122M_{\min} + 0.122. \quad (21)$$

Using (21) and (17) in (20), the input coupling network reflection coefficient magnitude $|s_{11}|$ has been computed as a function of frequency from 8 to 12 GHz. The corresponding locus of Y_{opm}^* is almost identical with Y_{om}^* which is given in [3]. In Fig. 5, $|s_{11}|$, normalized to G_{in}, and the equivalent VSWR is plotted against frequency. If the input network for each stage of the amplifier is designed with reflection coefficient less than or equal to $|s_{11}|$, then the specification on noise performance will be met. A convenient network in this case would be of the Chebyshev impedance-matching type with a maximum VSWR of 1.6 and designed to match into Y_{opm}^* across the amplifier passband [3].

In the second example, a single-stage amplifier is designed with an objective of amplifier noise figure equal to the minimum noise figure of the transistor at 12 GHz. The amplifier noise figure and the amplifier gain are both to be constant across the passband of 8-12 GHz. With $F = 4.5$ dB in (19), $|s_{21}|^2$ has been computed and is plotted against frequency in Fig. 6. The normalizing admittance for the input coupling network is Y_{of}^* which can be obtained from the plot of Y_{of} in [3]. To allow for gate bonding wire inductance, the series reactance of a 0.15-nH inductance is added to $Z_{of}^* = 1/Y_{of}^*$. This combination is modeled with an equivalent circuit consisting of the series connection of a 20-Ω resistor and an open-circuited length of transmission line of characteristic impedance 6.21 Ω and electrical length of 0.063 λ at 10 GHz.

The transducer power gain of the input network is approximated by a polynomial expression [8] which is prescribed to have a slope of 3 dB/octave over the range 6–12 GHz, rising to a value of 0 dB at 12 GHz. The approximated gain function is plotted in Fig. 6. The input network has been synthesized in distributed form using this function and, subject to gain–bandwidth limitations [9], is terminated with the equivalent circuit described in the previous paragraph. This equivalent circuit is omitted in the final amplifier realization. The complete amplifier is shown in Fig. 7. In Fig. 8 the transistor minimum noise figure $F_{\min}$ and the computed amplifier

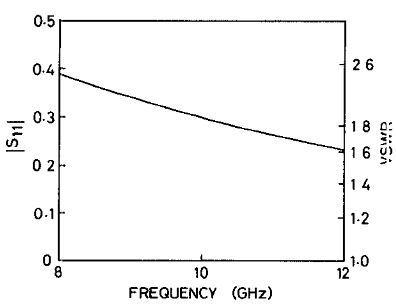

Fig. 5. Input coupling network reflection coefficient and VSWR for near-minimum noise measure design.

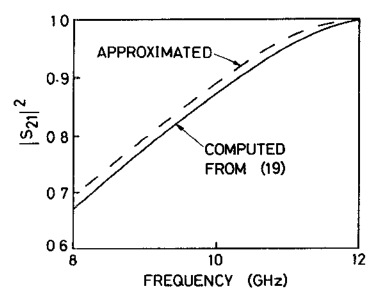

Fig. 6. Input coupling network transducer power gain for single-stage amplifier.

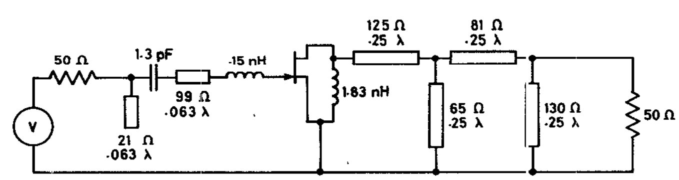

Fig. 7. Circuit of single-stage amplifier. Transmission lines are specified by characteristic impedance and electrical length at 10 GHz.

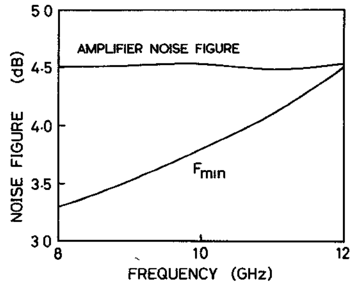

Fig. 8. Minimum noise figure of the transistor and noise figure of the single-stage amplifier.

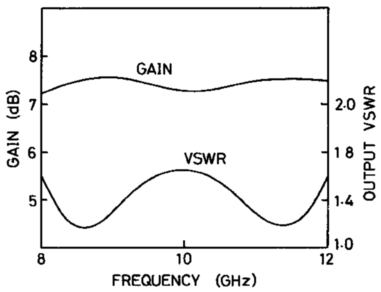

Fig. 9. Gain and output VSWR of single-stage amplifier.

noise figure are plotted against frequency. The amplifier noise figure is close to 4.5 dB across the passband as expected. It is worth noting that the approximated function of Fig. 6 represents a good impedance match between the input coupling network and Y_{of}^* at 12 GHz, but at other frequencies the function falls off in magnitude and the match is degraded. The gain–bandwidth limitations associated with the function are therefore considerably less severe than for a Chebyshev function which approximates a good match across the entire passband. This property of the approximated function permits a lower amplifier noise figure at 12 GHz than is generally available with a Chebyshev coupling network, but the improvement at 12 GHz is achieved at a cost of increased noise figure at lower frequencies.

As well as producing the prescribed noise figure characteristic, the input network affects the available gain of the transistor. For this example, the available gain of the transistor is almost constant across the amplifier passband. The lossless output matching network in Fig. 7 has been designed by a direct synthesis technique [8] using a Chebyshev gain characteristic with 0.25 dB of ripple. The computed amplifier gain and output VSWR is shown in Fig. 9.

IV. CONCLUSION

The loci of constant overall noise figure derived in this short paper can be used to graphically investigate the noise performance of an amplifier for various preamplifier source admittances and main amplifier noise figures. This has been illustrated with an example. An input coupling network synthesis method has been described for noise figure and noise measure specifications. The method enables a wide variety of broad-band design problems to be handled since it reduces the low-noise design problem to an impedance-matching problem which can be solved by well-known analytical techniques. Advantages of the method are related to its simplicity compared with existing low-noise design methods and to the information it can provide regarding noise figure limitations for a given transistor and bandwidth.

REFERENCES

[1] "IRE standards on methods of measuring noise in linear two ports, 1959," *Proc. IRE*, vol. 48, pp. 60–68, Jan. 1960.

[2] H. Fukui, "Available power gain, noise figure, and noise measure of two-ports and their graphical representations," *IEEE Trans. Circuit Theory*, vol. CT-13, pp. 137–142, June 1966.

[3] C. A. Liechti and R. L. Tillman, "Design and performance of microwave amplifiers with GaAs Schottky-gate field-effect transistors," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 510–517, May 1974.

[4] W. Baechtold and M. J. O. Strutt, "Noise in microwave transistors," *IEEE Trans. Microwave Theory Tech.* (*Special Issue on Noise*), vol. MTT-16, pp. 578–585, Sept. 1968.

[5] K. Hartmann and M. J. O. Strutt, "Changes of the four noise parameters due to general changes of linear two-port circuits," *IEEE Trans. Electron Devices*, vol. ED-20, pp. 874–877, Oct. 1973.

[6] K. Hartmann and M. J. O. Strutt, "Scattering and noise parameters of four recent microwave bipolar transistors up to 12 GHz," *Proc. IEEE* (Lett.), vol. 61, pp. 133–135, Jan. 1973.

[7] J. Lange, "Noise characterization of linear two ports in terms of invariant parameters," *IEEE J. Solid-State Circuits*, vol. SC-2 pp. 37–40, June 1967.

[8] R. S. Tucker, "Synthesis of broadband microwave transistor amplifiers," *Electron. Lett.*, vol. 7, pp. 455–456, Aug. 12, 1971.

[9] D. C. Youla, "A new theory of broad-band matching," *IEEE Trans. Circuit Theory*, vol. CT-11, pp. 30–50, Mar. 1964.

Bias Frequency Modulation of GaAs Millimeter-Wave Diode Oscillators

M. J. LAZARUS, MEMBER, IEEE,
K. Y. CHEUNG, STUDENT MEMBER, IEEE,
S. NOVAK, AND A. M. SOKULLU

***Abstract*—A method for estimation of FM bias modulation sensitivity of a Gunn-diode oscillator is presented under the assumption of incomplete domain formation for a very short diode. Experimental measurements at 33 and 11.3 GHz are shown as compared with this estimate and the decrease of sensitivity with external Q, for modulation frequency high enough to eliminate thermal effects, is demonstrated.**

I. INTRODUCTION

Space-charge waves in gallium arsenide (GaAs) have been considered in terms of an elegant analysis [1] by fitting the best experimental measurements of electron velocity v, as a function of electric field E [2], to the expression $1/v = A + BE$, with empirical choice of constants A and B. Experimental results [1] for the rate of change of phase delay θ of the space-charge wave with respect to variation of dc voltage V, show that $\partial\theta/\partial V$ is approximately $B\omega$ for microwave frequency $\omega/2\pi$, until biasing fields exceed 10^6 V/m. For higher fields, velocity saturation causes decrease from $B\omega$.

In this short paper, these effects are related to the phase of the device impedance or admittance under the assumption that the space-charge growth is *small* for short lightly doped samples used in oscillators. A formula is then developed which gives an estimate of the sensitivity of the oscillator to direct frequency modulation (FM) by perturbation of the bias voltage, once the Q of the oscillator is known. Throughout this work, we do not distinguish between loaded Q_L and external Q_{ext}, since the unloaded Q_U was found experimentally to be orders of magnitude higher. Hence we assume $1/Q_L = 1/Q_U +$

Manuscript received May 28, 1974; revised March 13, 1975. This work was supported by the Science Research Council, England, under Grant B/RG/317.

M. J. Lazarus is with the Department of Physics, University of Lancaster, Lancaster, England. He is presently on sabbatical leave at the Department of Physics, University of California at Davis, Davis, Calif. 95616.

K. Y. Cheung, S. Novak, and A. M. Sokullu are with the Department of Physics, University of Lancaster, Lancaster, England.